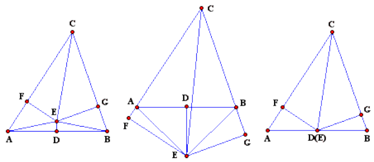


**Exercise 1.** Proof: Draw an arbitrary triangle ABC with AC¿BC. I plan to prove that AC=BC. Bisect the angle C, and draw the perpendicular bisector of side AB (bisecting AB at D). These two lines (the bisector of angle C, and the perpendicular bisector of CD) cannot be the same line or be parallel, as then the triangle ABC would be isosceles. So they must meet at some point E. This point E must be either inside the triangle, or outside the triangle, or on the line segment AB. I have drawn all three possible cases, on the left above. In the first two cases, draw the lines AE and BE. In all three cases draw the lines EF and EG, perpendicular to sides AC and BC respectively, as shown in the diagram. Let's examine all three cases.

Case I: Right triangles CEF and CEG are congruent. So, EF=EG and CF=CG. Right triangles ADE and BDE are congruent. So AE=BE. And right triangles AEF and BEG are congruent. So AF=BG. And AC=BC by adding of equals to equals. In other words, triangle ABC is isosceles.

Case II: The same exact argument (except that at the end we subtract equals from equals) shows that this too is isosceles.

Case III: This one is slightly easier, as there are fewer triangles. But the argument is similar.

Triangle ABC is isosceles, in all three cases.

**Theorem 0.1.** *Area of Triangle:* $\frac{1}{2}bh$
*Your most basic form of triangle area finding.*

**Theorem 0.2.** *Area of Triangle:* $\frac{1}{2}ab\sin\theta$
*Derived from the previous formula, where $\theta$ is the angle between the sides $a$ and $b$.*

**Theorem 0.3.** *Area of Triangle:* $\sqrt{s(s-a)(s-b)(s-c)}$
*Where $s = \frac{a+b+c}{2}$, and $a$, $b$, $c$, are the sides; $s$ is also known as the semiperimeter.*
*This is called Heron's Formula.*

**Theorem 0.4.** *Area of Triangle: Shoelace Theorem*
*Just read up here:* `http://www.artofproblemsolving.com/Wiki/index.php/Shoelace_Theorem`
*Honestly you won't see much of this. But is useful occassionally.*

**Theorem 0.5.** *Law of Cosines:* $a^2 + b^2 - 2ab\cos\theta = c^2$
*Where $\theta$ is opposite side $c$. Your geometry swiss army knife.*